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FABRY-PEROT RESONANCES AT SMALL FRESNEL NUMBERS

S. R. Barone and M. C. Newstein

TRG Incorporated
A Subsidiary of
Control Data Corporation
Route 110
Melville, New York

Contract No. AF 19(604)-8817

Project No. 4645

Task No. 464502

Scientific Report No. 3

January 1965

Prepared for:

AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS

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ABSTRACT

A theory of Fabry-Perot resonances designed to be useful at small Fresnel numbers is given. This theory is applied to the case where the interferometer mirrors are of arbitrary curvature and have circular apertures. Asymptotic formulas are derived for the diffraction output and phase shifts in the limit of small Fresnel numbers. These formulas demonstrate the mode discrimination properties of the interferometers when operated in this regime. Numerical results are presented covering a wider range of Fresnel numbers.

INTRODUCTION

The resonance properties of Fabry-Perot (F.P.) interferometers at Fresnel numbers of the order of, and greater than, unity have been treated in a number of papers.[1-6] The recent use of "diffraction output coupling" as an efficient method of transverse mode suppression when the Fresnel numbers are less than unity[7] has motivated us to extend the theory so as to describe resonators operating in this range. We have previously presented numerical results for small Fresnel numbers, on diffraction outputs and phase shifts for F.P. interferometers consisting of plane mirrors with circular apertures.[8]

In this article we give a theory of F.P. resonances designed to be useful at small Fresnel numbers. This theory is applied to the case where the interferometer mirrors are of arbitrary (small) curvature and have circular apertures. Asymptotic formulas are derived for the diffraction output and phase shifts in the limit of small Fresnel numbers. These formulas demonstrate the mode discrimination properties of the interferometers when operated in this regime. Numerical results, in addition to those previously reported[8], are presented on the diffraction output and phase shifts for curved circular mirrors for small Fresnel numbers. The theory is presented in such a way (in terms of a canonical integral equation) that it applies to a general class of aperture shapes.

FORMULATION OF THE PROBLEM

The geometry which is considered consists of a symmetric pair of mirrors, each of radius of curvature, R, whose subtending planes are separated along the axis of the resonator by a distance L. The aperture shapes are taken to be circles of radius a*. (See Figure 1). We are interested in the resonance properties of this electromagnetic cavity, in particular, the complex resonant frequencies** and the corresponding field configurations. The information contained in the complex resonant frequencies can be expressed in terms of the single pass diffraction output and phase shift of the resonant fields.

Consider an arbitrary field configuration at plane (1) of Figure 1. This field may be regarded as the sum of two fields, one moving to the right, ${}^{(1)}f_+(x,y)$, and one to the left ${}^{(1)}f_-(x,y)$. The field moving to the right reflects from the other mirror and results in a configuration ${}^{(2)}f_-(x,y)$, representing a field moving to the left at plane (2). The resonances of this open structure are defined by the requirement that after a double transit the field reproduces itself, or, after a single pass it reproduces itself up to the sign change ± 1, that is

$$ {}^{(2)}f_-(x,y) = \pm\, {}^{(1)}f_-(x,y) \tag{1} $$

More generally, we have:

$$ {}^{(2)}f_-(x,y) = re^{iK(k)L} \quad {}^{(1)}f_-(x,y), \tag{2} $$

where K(k)L is a complex number, independent of x and y, but a function of the geometry and propagation constant and r is the mirror reflectivity. The single pass phase shift, ϑ, is given by:

* The method of analysis can be extended to other aperture shapes, in particular, rectangles.

** Because the structure is open, the resonant frequencies are complex. The imaginary part of the frequency is the inverse of the lifetime of the corresponding "mode".

$$\phi = \mathrm{Re}\ K(k)L, \tag{3}$$

and the single pass fractional power output (due to diffraction), $1 - |\gamma|^2$, is determined by

$$|\gamma|^2 = \left|e^{iK(k)L}\right|^2 \tag{4}$$

The resonant values of the frequency, Re ck, and gain coefficient, Imk, are determined by the resonance condition (1), or

$$re^{iK(k)L} = \pm 1.$$

The quantity of interest, K(k), is determined by the solution of the integral equation which is obtained by propagating the field from plane (1) to plane (2) and using equation (2). The following approximations allow a relatively simple formulation of the problem:

(1) The mirror radius of curvature, R, is large enough that it is consistent to take the electric (or magnetic) field vector to be always perpendicular to the resonator axis. This reduces the real situation to a scalar problem where the total field is determined by a single transverse component.

(2) For a field on a plane intersecting the rim of a mirror (such as plane (1) or (2) in Figure 1) moving toward the mirror, the reflected field moving away from the mirror, at the point (x,y) on this plane is:

(a) zero, for $x^2 + y^2 > a^2$ (Kirchoff-approximation,

(b) equal in amplitude to the reflection coefficient, r, times the incident field, and increased in phase factor by the amount $\exp\left[2\,i\,\frac{a^2}{R}\left(1 - \frac{x^2+y^2}{a^2}\right)\right]$

for $x^2 + y^2 \leq a^2$. This phase is the amount that would be accumulated by a ray moving parallel to the axis.

(c) $(L/a)^2 \gg (a^2/L\lambda)$ = Fresnel number.

Because of the axial symmetry the solutions must be of the form:

$$f(x,y) = R^n(\rho)e^{in\theta}, \text{ where}$$

$$x = a\rho \cos \theta$$

$$y = a\rho \sin \theta.$$

Using assumptions (a), (b) and (c) and the resonance condition (1), we obtain the integral equation:*

$$\gamma_{n,m} R_{n,m}(\rho) = \int_0^1 \rho' d\rho' \; R_{n,m}(\rho') e^{i\pi N\rho^2} e^{i\pi N[1-2L/R]\rho'^2} J_n(2\pi N\rho\rho') \tag{5}$$

The propagation factors $\exp(iK_{n,m}L)$ are obtained from the eigenvalues $\gamma_{n,m}$ of equation 5 by the relation:

$$\exp(iK_{n,m}L) = \gamma_{n,m} \, 2\pi N \exp[ikL - i\tfrac{\pi}{2}(n+1)]. \tag{6}$$

In the above relations, the Fresnel number, N, is given by:

$$N = a^2/L\lambda \tag{7}$$

Canonical Integration Equation

The integral equation (5) may be identified with a canonical form:

$$\gamma\psi(x) = \int_a^b \phi(x) \, K(\gamma xx') \, \phi(x') \, \psi(x') \, \omega(x') \, dx'. \tag{8}$$

This same form arises when the aperture shape corresponds to separable coordinate systems other than circular, in particular, rectangular. For the specific case of circular apertures the

*See Appendix A.

particular identifications that are necessary in order to convert the integral equation (5) into the form (8) are:

$$x = \rho^2, \tag{9a}$$

$$K(\xi) = n!\ \xi^{-n/2}\ J_n(2\sqrt{\xi})$$
$$= 1 - \frac{\xi}{1!(n+1)} + \frac{\xi^2}{2!(n+1)(n+2)} \cdots\cdot, \tag{9b}$$

$$\psi(x) = x^{-\frac{n}{2}}\ R_n(x)\ \exp[-i\pi NLx/R], \tag{9c}$$

$$\Gamma = 2n!\ \gamma_{n,m}/(\pi N)^n, \tag{9d}$$

$$\phi(x) = \exp[i\pi Nx(1-L/R)], \tag{9e}$$

$$\omega(x) = x^n, \tag{9f}$$

$$\lambda = (\pi N)^2, \tag{9g}$$

$$a = 0,\ b = 1. \tag{9h}$$

Two methods for obtaining solutions to equation (8) will be presented. One is particularly convenient for getting the asymptotic behavior in the limit of small λ and expansions in power series of the Fresnel number useful for $\pi N < 1$. The other is useful as a basis for machine computations in order to obtain the resonance properties for values of the Fresnel number up to unity.

ASYMPTOTIC SOLUTIONS

In this section we investigate the eigenvalues, Γ, and the eigenfunctions, $\psi(x)$, of the canonical integral equation (8) in the limit $\eta \rightarrow 0$. Correspondence will be made with the asymptotic properties of the resonances of the circular aperture F.P. interferometer in the limit $N \rightarrow 0$, by means of the relations (9a) to (9h).

The eigenvalues Γ, eigenfunctions $\psi(x)$ and the kernal $K(\eta xx')$ are expanded in power series in the parameter η:

$$\Gamma = \sum_{m=o}^{\infty} \Gamma_m \eta^m, \tag{10}$$

$$\psi(x) = \sum_{m=o}^{\infty} \psi_m(x) \eta^m, \tag{11}$$

and

$$K(\eta xx') = \sum_{m=o}^{\infty} k_m \, x^m \, x'^m \, \eta^m. \tag{12}$$

When these three expansions are substituted into the integral equation (8), we obtain, upon equating coefficients of equal powers of η:

$$\sum_{m=o}^{s} \psi_m(x) \, \Gamma_{s-m} = \phi(x) \sum_{m=o}^{s} k_m \, x^m \, C_{s,m}, \tag{13a}$$

where

$$C_{s,m} = \int dx' \, \psi(x') \, x'^m \, \phi(x') \, \psi_{s-m}(x'), \tag{13b}$$

and

$$s = 0, 1, 2, \ldots \quad . \tag{13c}$$

We observe, from equations (13) that either $\psi_m(x)$ is of the form:

$$\psi_m(x) = \phi(x) \cdot \text{(Polynomial of degree m)},$$

or $\Gamma_o = 0$. Calling the first solution ${}^{(0)}\psi_{m(x)}$, and the second ${}^{(1)}\psi_{m(x)}$, we find, either ${}^{(1)}\psi_{m(x)}$ is of the form:

$${}^{(1)}\psi_{m(x)} = \phi(x) \cdot \text{(Polynomial of degree m + 1)},$$

or $\Gamma_1 = 0$. Continuing in this way we see that there are an infinite number of modes ${}^{(i)}\psi_{(x)}$, with the corresponding eigenvalues ${}^{(i)}\Gamma$, which have the expansions:

$${}^{(i)}\psi_{(x)} = \sum_{m=o}^{\infty} {}^{(i)}\psi_{m(x)} \, \eta^m, \tag{14}$$

$${}^{(i)}\psi_{m(x)} = \phi(x) \sum_{\ell=o}^{m=i} {}^{(i)}p_{m,\ell} \, x^{\ell}, \tag{15}$$

$${}^{(i)}\Gamma = \sum_{m=o}^{\infty} {}^{(i)}\Gamma_m \, \eta^{m+i} \tag{16}$$

Substituting these relations into the integral equation (8) and equating coefficients of equal powers of η, and then equating coefficients of equal powers of x, we finally obtain the relations:

$$\sum_{n=o}^{s-m} {}^{(i)}\Gamma_n \, {}^{(i)}p_{s-i-n,m} = k_m \sum_{n=o}^{s-m+i} {}^{(i)}p_{s-m,n} \, t_{n+m}, \tag{17}$$

where

$$t_m = \int_a^b \phi^2(x) x^m \rho(x) \, dx, \tag{18}$$

$$p_{n,m} \equiv 0 \text{ for n negative} \tag{19}$$

$$s = 0, 1, 2, \ldots \tag{20}$$

$$m = 0, 1, 2, \ldots s-1, s. \tag{21}$$

Equations (17) through (21) completely determine the solutions since they determine the quantities ${}^{(i)}p_{m,\ell}$ and ${}^{(i)}\lambda_m$ which through equations (14), (15) and (16) give the eigenfunctions and eigenvalues. The quantities t_m and k_m are known. The wave functions are determined up to an arbitrary normalization factor. A convenient normalization, which is used below, is:

$$\psi(0) = \phi(0). \tag{22}$$

This normalization implies:

$$ {}^{(i)}P_{m,0} = \delta_{n,0} \; . \tag{23}$$

The coefficients necessary in order to get the first two terms in the λ expansions of ${}^{(i)}\psi(x)$ and ${}^{(i)}\lambda$ are, for the cases $i = 0$, and $i = 1$:

CASE 1 $i = 0$

$$ {}^{0}\lambda_0 = k_0 t_0 \tag{24a}$$

$$ {}^{0}\lambda_1 = k_1 t_1^{2}/t_0 \tag{24b}$$

$$ {}^{0}P_{11}/{}^{0}P_{00} = k_1 t_1/k_0 t_0 \tag{24c}$$

CASE 2 $i = 1$

$$ {}^{1}\lambda_0 = \frac{k_1}{t_0}\,(t_2 t_0 - t_1^{2}) \tag{25a}$$

$$ ({}^{1}P_{01}/{}^{1}P_{00}) = -\, t_0/t_1 \tag{25b}$$

$$ ({}^{1}P_{12}/{}^{1}P_{00}) = \frac{k_2}{P_0 t_1}\,(t_2 t_1 - t_3 t_0) \tag{25c}$$

$$ ({}^{1}P_{11}/{}^{1}P_{00}) = \frac{{}^{1}\lambda_0}{t_1 k_0} - \left(\frac{{}^{1}P_{12}}{{}^{1}P_{00}}\right)\frac{t_2}{t_1} \tag{25d}$$

$$ {}^{1}\lambda_1 = k_1\left[\frac{{}^{1}P_{11}}{{}^{1}P_{01}}\, t_2 + \frac{{}^{1}P_{12}}{{}^{1}P_{01}}\, t_3\right] - {}^{1}\lambda_0\,\frac{{}^{1}P_{11}}{{}^{1}P_{01}} \tag{25e}$$

We now specialize to the case of circular apertures. Using the relations (9a) - (9h), and equation (6), we have, for the propagation factor:

$$\exp i\left[K_{n,m}\,L\right] =$$

$$\frac{(\pi N)^{n+2m+1}}{n!}\,{}^{m}\Gamma_{o}\,\exp\left[ikL - i\tfrac{\pi}{2}(n+1) + 2\pi i NL/R\right] \tag{26}$$

$$\left[1 + O(N^2)\right]$$

The quantity ${}^{m}\Gamma_{o}$ is determined by equation (17) as a function of the k's and t's. The latter quantities are given, asymptotically, by:

$$t_j = \frac{1}{n-j+1}\,\exp\left[i2\pi(1-L/R)N\left(\frac{n+j+1}{n+j+2}\right)\right] + O(N^2) \tag{27}$$

The former are given by:

$$k_j = (-1)^j\, n!/(j!)\,(n-j)! \tag{28}$$

As the Fresnel number gets very small, the quantity ${}^{m}\Gamma_{o}$ approaches a constant (independent of N) and hence the dependence on the Fresnel number of the single pass diffraction loss is determined by:

$$\left|\exp\left[i\,K_{n,m}\,L\right]\right|^2 \underset{N\to 0}{\sim} N^{2n+4m+2} \tag{29}$$

This relation illustrates the strong mode discrimination in the limit of small Fresnel numbers.

MORE GENERAL EXPANSIONS

The complete expansion in powers of the Fresnel number for the lowest modes (m=0) for any value of n can be obtained from simple recurrence relations. Thus, we have from equation (17) for i = 0, m = 0

$$\lambda_s = \frac{k_o}{p_o} \sum_{n=o}^{s} p_{s,n}\, t_n, \tag{30}$$

where

$$p_{ss} = \frac{k_s}{k_o}\,\frac{t_s}{t_o}\, p_{oo}, \tag{31}$$

and the quantities $p_{s,n}$ for $n < s$ are related to previously determined quantities by the expression:

$$p_{s,n} = \frac{k_n}{\lambda_o} \sum_{j=o}^{s-n} p_{s-n,j}\, t_{j-n} - \frac{1}{\lambda_o} \sum_{j=1}^{s-n} \lambda_j\, p_{s-j,n}, \tag{32}$$

which is obtained from equation 17 for i=0, by setting m=n.

The corresponding complete expansion for the higher modes ($i > 0$) cannot be obtained in terms of simple recurrence relations, but rather require the solution of simultaneous algebraic equations. In order to obtain numerical results for the case that the Fresnel number is not much less than unity an alternative method of analysis to that described above may be employed. This method treats groups of modes on an equal basis and hence while not as satisfactory for the lowest mode is more easily mechanized for machine computation.

Referring to the canonical integral equation (8), for the Mth degree of approximation we represent the kernal $K(\xi)$, by the Mth degree polynomial:

$$K_m(\xi) = \sum_{n=o}^{M} k_n\, \xi^n . \tag{33}$$

The integral equation becomes:

$$\lambda \psi(x) = \sum_{n=o}^{M} k_n x^n \mu^n \phi(x) \int_a^b \phi(x') \psi(x') \rho(x') dx' . \tag{34}$$

The solutions of this integral equation must be of the form:

$$\psi(x) = \phi(x) \sum_{n=o}^{M} P_n X^n . \tag{35}$$

The coefficients P_n are determined by substituting (35) into (34) and equating coefficients of equal powers of x. In this way one gets the M+1 homogeneous equations for the M+1 unknowns $P_o, \cdots P_M$:

$$\lambda P_n = \sum_{m=o}^{M} k_n \mu^n t_{n-m} P_m , \tag{36}$$

where, as before:

$$t_n = \int_a^b \phi^2(x)\ x^n\ \rho(x) dx . \tag{37}$$

The M+1 eigenvalues λ, are solutions of the determintal equation:

$$\left| k_n \mu^n t_{n-m} - \lambda \right| = 0 \tag{38}$$

This method of solution has been described by She and Heffner (9).

RESULTS

From equations (24) to (28) for the asymptotic propagation factors for the first four modes are given by:

$$\exp[i K_{o,o} L] = (\pi N) \exp i[\Phi - \frac{\pi}{2} + \pi N g] \tag{39a}$$

$$\exp[i K_{1,o} L] = \frac{1}{2}(\pi N)^2 \exp i[\Phi - \pi + \frac{4}{3} \pi N g] \tag{39b}$$

$$\exp[i K_{o,1} L] = \frac{1}{12}(\pi N)^3 \exp i[\Phi - \frac{\pi}{2} - \pi N g] \tag{39c}$$

$$\exp[i K_{1,1} L] = \frac{1}{36}(\pi N)^4 \exp i[\Phi - \frac{16}{15} \pi N g], \tag{39d}$$

where Φ is the geometrical phase shift given by

$$\Phi = kL - 2\pi NL/R, \tag{39e}$$

and

$$g = 1 - L/R. \tag{39f}$$

It is generally true that the phase shift, relative to the geometrical phase shift is linear in the quantity Ng asymptotically for small enough N. As was mention above, the asymptotic dependence on the Fresnel number of the single pass diffraction loss is determined by:

$$\left|\exp[i K_{n,m} L]\right|^2 \underset{N \to o}{\sim} N^{2n-4m-2} \tag{40}$$

The results for a wider range of Fresnel numbers is illustrated in figures (2) to (5). In figures (2) and (3) we have plotted $|\gamma|^2 = |\exp[i KL]|^2$ for the first two modes, (n=0, m=0) (n=1, m=0) respectively, versus the Fresnel number $a^2/L\lambda$. The reciprocal of $|\gamma|^2$ is the single pass gain required to reach the threshold of oscillation in the mode. The fractional diffracted power output per transit is $1 - |\gamma|^2$. The quantity $g = (1 - L/R)$ measures the mirror curvature. The case $g = 0$

corresponds to a confocal arrangement and the case $g = 1$ to flat mirrors. For small enough values of the Fresnel number the losses are insensitive to the value of g. The detailed dependence on g for the larger values of N is given in expanded scale in the lower right hand corners of figures (2) and (3). It is seen, that for the losses, the asymptotic expressions (39) become quite accurate for Fresnel numbers less than 0.1 for the lowest mode and 0.2 for the first excited mode.

In figures (4) and (5) we have plotted the phase shift (relative to the geometrical phase shift versus the Fresnel number for the first two modes. It is seen that the asymptotic expressions (39) give the phase shift quite accurately for Fresnel numbers less than 0.1.

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APPENDIX A

DERIVATION OF RESONANCE INTEGRAL EQUATION

If a field distribution is given over the plane $z'=0$, then the field at the point (x,y,L), which satisfies the scalar Helmoltz equation is:

$$f(x,y,L) = \int dx'dy' \; f(x',y',z') \left.\frac{\partial}{\partial z'} G(x,y,L,x',y',z')\right|_{z'=0} \qquad \text{(A-1)}$$

where $G(x,y,L;\, x',y',z')$ is the Greens function for the scalar Helmoltz equation which vanishes in the plane $z'=0$ and represents outgoing waves. This Greens function is given by:

$$G(x,y,L,\, x',y',z') = \frac{1}{4\pi}\left[\frac{e^{ik\sqrt{(x-x')^2+(y-y')^2+(L-z')^2}}}{\sqrt{(x-x')^2+(y-y')^2+(L-z')^2}} - \frac{e^{ik\sqrt{(x-x')^2+(y-y')^2+(L+z')^2}}}{\sqrt{(x-x')^2+(y-y')^2+(L+z')^2}}\right] \qquad \text{(A-2)}$$

Thus, equation (1) may be written

$$f(x,y,L) = \frac{-1}{2\pi} \; ikL \int dx'dy' \; f(x',y',o) \; \frac{e^{ikR}}{R^2}, \qquad \text{(A-3)}$$

where $R = \sqrt{(x-x')^2 + (y-y')^2 + L^2}$.

If the mirror diameter, 2a is small compared to the distance L, then we may make the expansion:

$$R \approx L\left[1 + \frac{1}{2}\frac{(x-x')^2+(y-y')^2}{L^2} + 0\left(\frac{a^4}{L^4}\right)\right].$$

In the argument of the exponential term, the dropping of the remainder $0\left(\frac{a^4}{L^4}\right)$ is justified if

$$kL\ 0\left(\frac{a^4}{L^4}\right) << -$$

This implies the requirement:

$$N = \frac{a^2}{L\lambda} << \left(\frac{L}{a}\right)^2 \qquad \text{(A-4)}$$

To get the form of equation (5) we must include the additional phase shift and the reflectivity of the mirrors. Using assumption (2) of the section "Formulation of the Problem" we have

$$f_+^{(2)}(\rho,\theta) = \frac{re^{ikL-i\pi/2}a^2}{[L+a^2/R]\lambda}\int_0^1 \rho' d\rho' \int_0^{2\pi} d\theta'\ f_-^{(1)}(\rho',\theta')\exp(-2\pi i\frac{NL}{R}\rho'^2)$$

$$\exp i\frac{\pi a^2}{[L+a^2/R]\lambda}\left[\rho^2 - \rho'^2 - 2\rho\rho'\cos(\theta-\theta')\right].$$

(A-5)

Using the relation:

$$2\pi(-i)^n J_n(z) = \int_0^{2\pi} du\ e^{inu}\ e^{iz\cos a}, \qquad \text{(A-6)}$$

and taking $N << R/\lambda$, we get the form of equation (5).

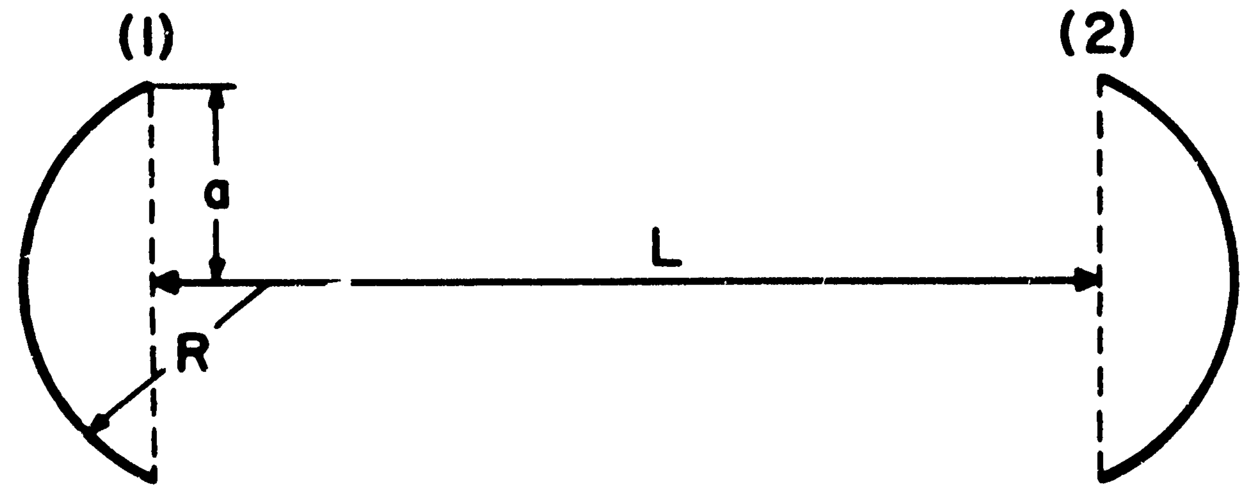


FIGURE 1. ILLUSTRATING RESONATOR CONFIGURATION

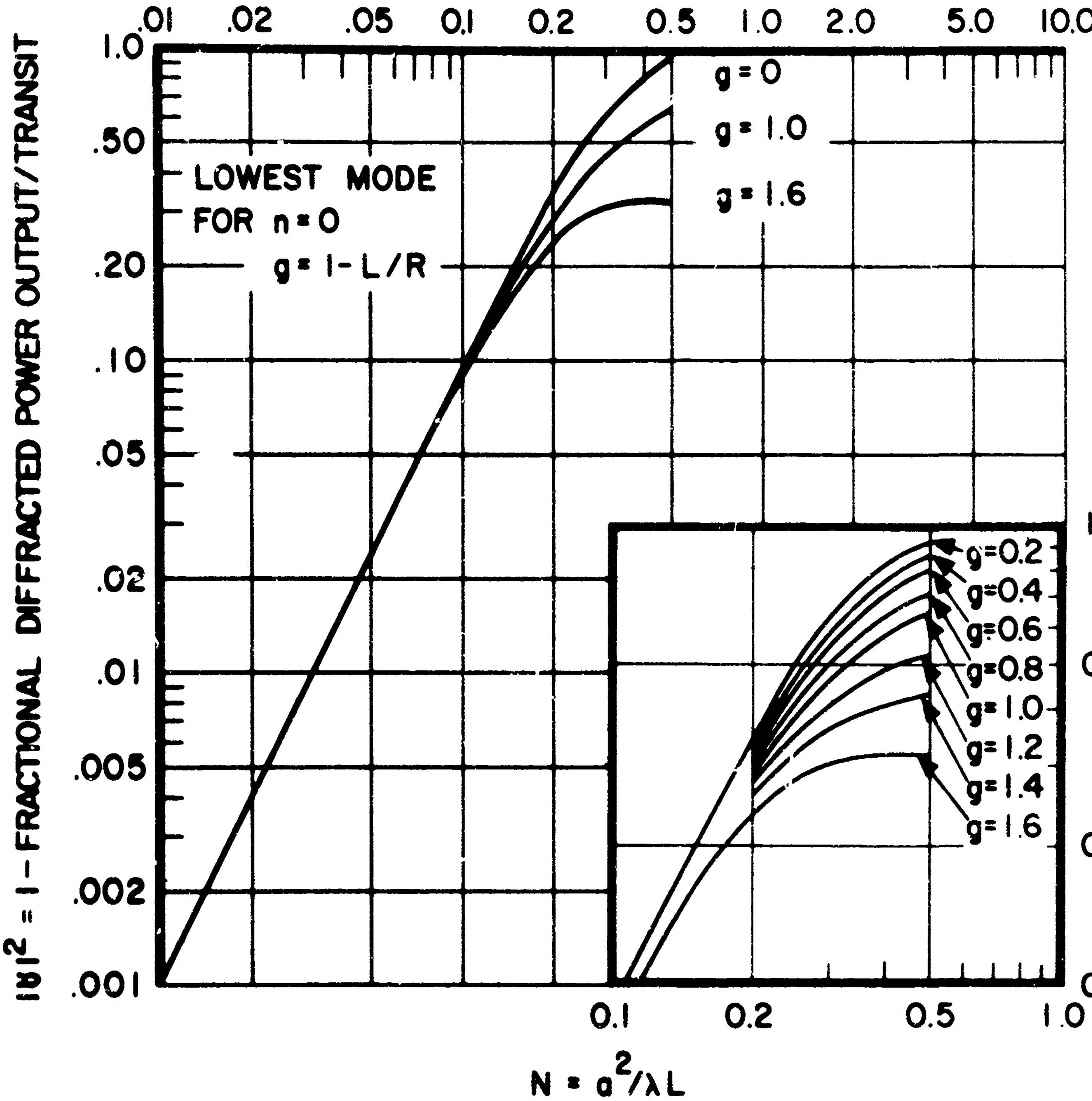
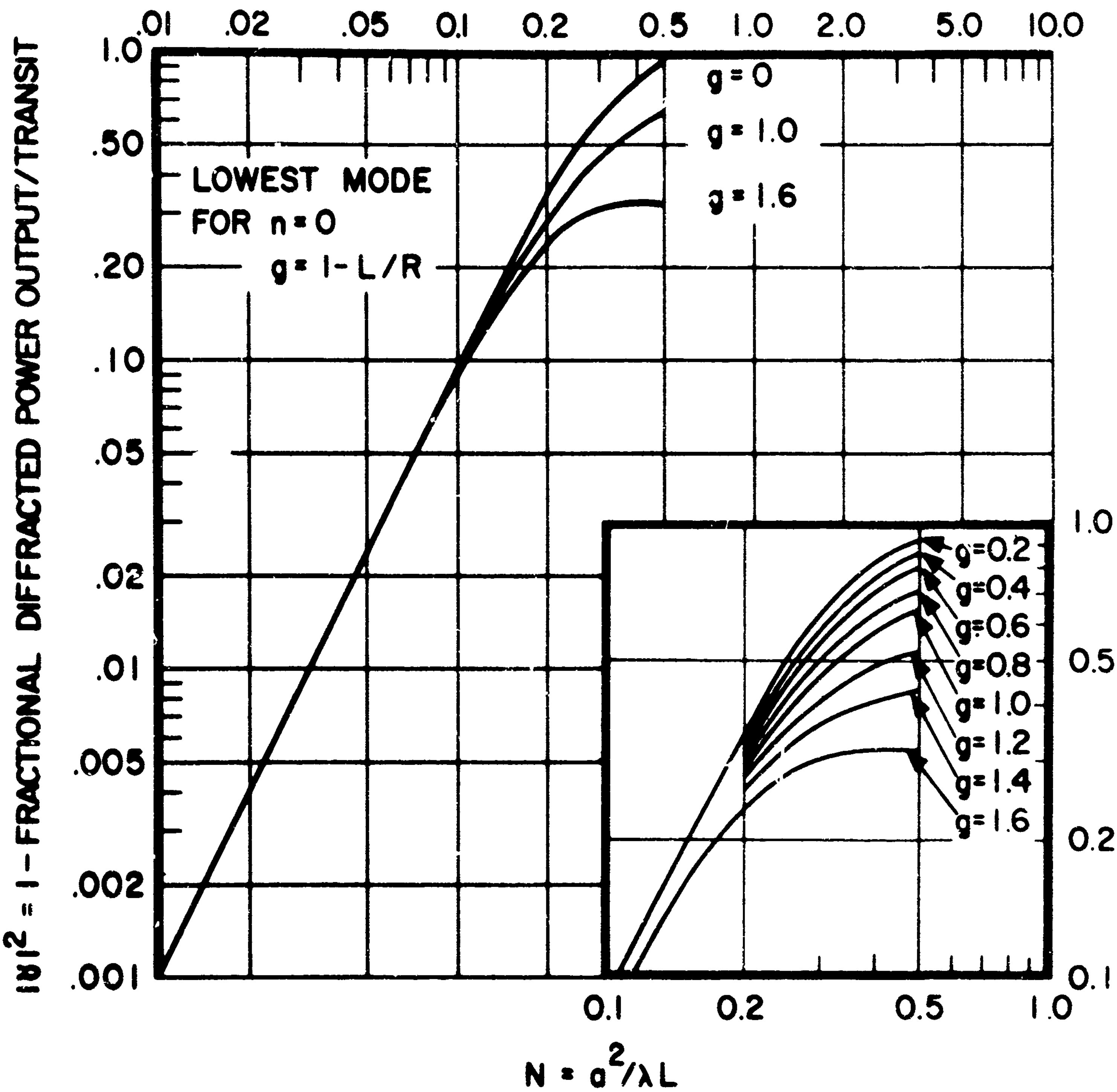


FIGURE 2

FRACTIONAL DIFFRACTED OUTPUT POWER PER TRANSIT vs.
FRESNEL NUMBER (LOWEST MODE FOR n=0)



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FRACTIONAL DIFFRACTED OUTPUT POWER PER TRANSIT vs.
FRESNEL NUMBER (LOWEST MODE FOR n=0)

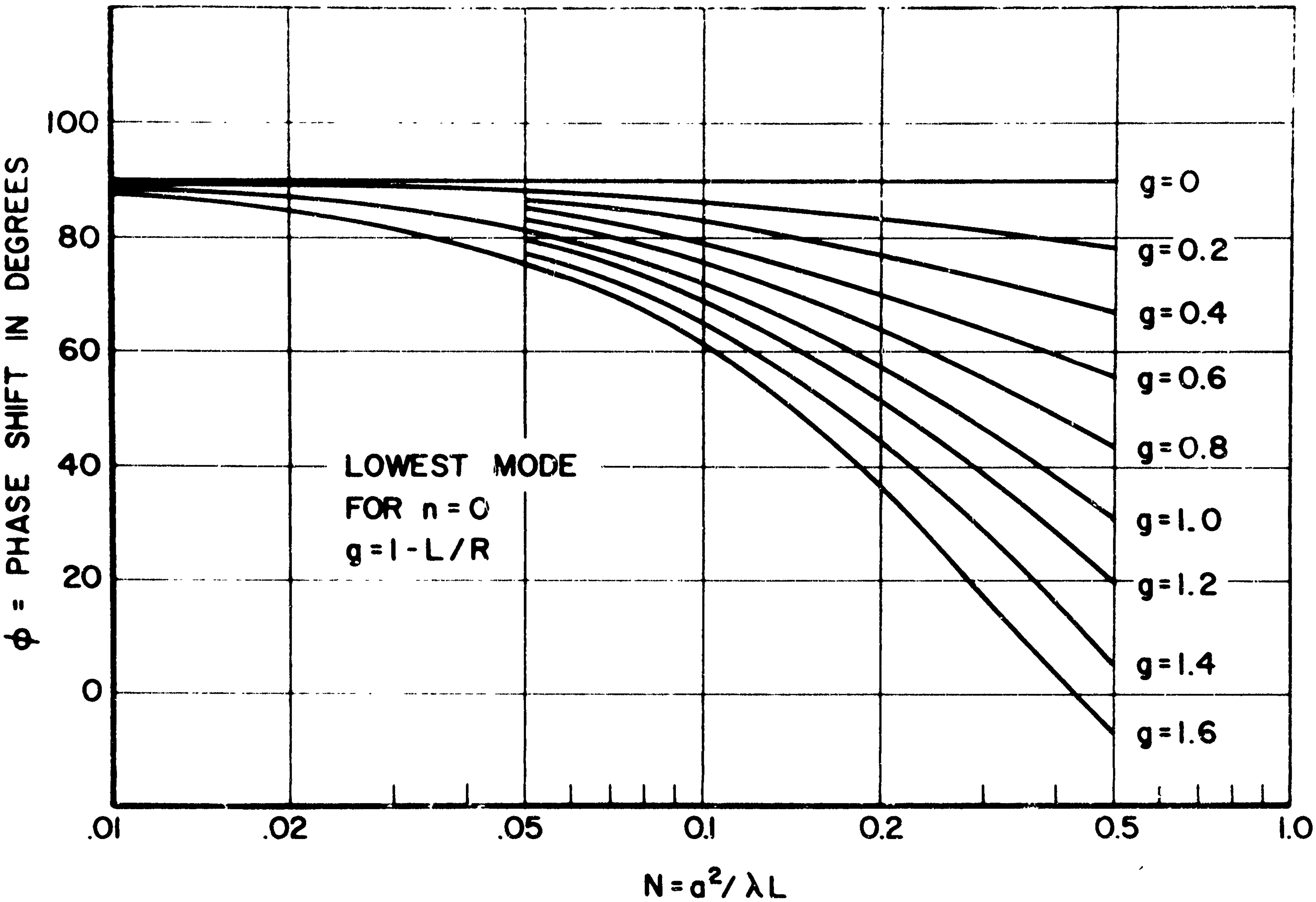

FIGURE 4

PHASE SHIFT PER TRANSIT (RELATIVE TO GEOMETRICAL PHASE SHIFT) vs. FRESNEL NUMBER (LOWEST MODE FOR n=0)

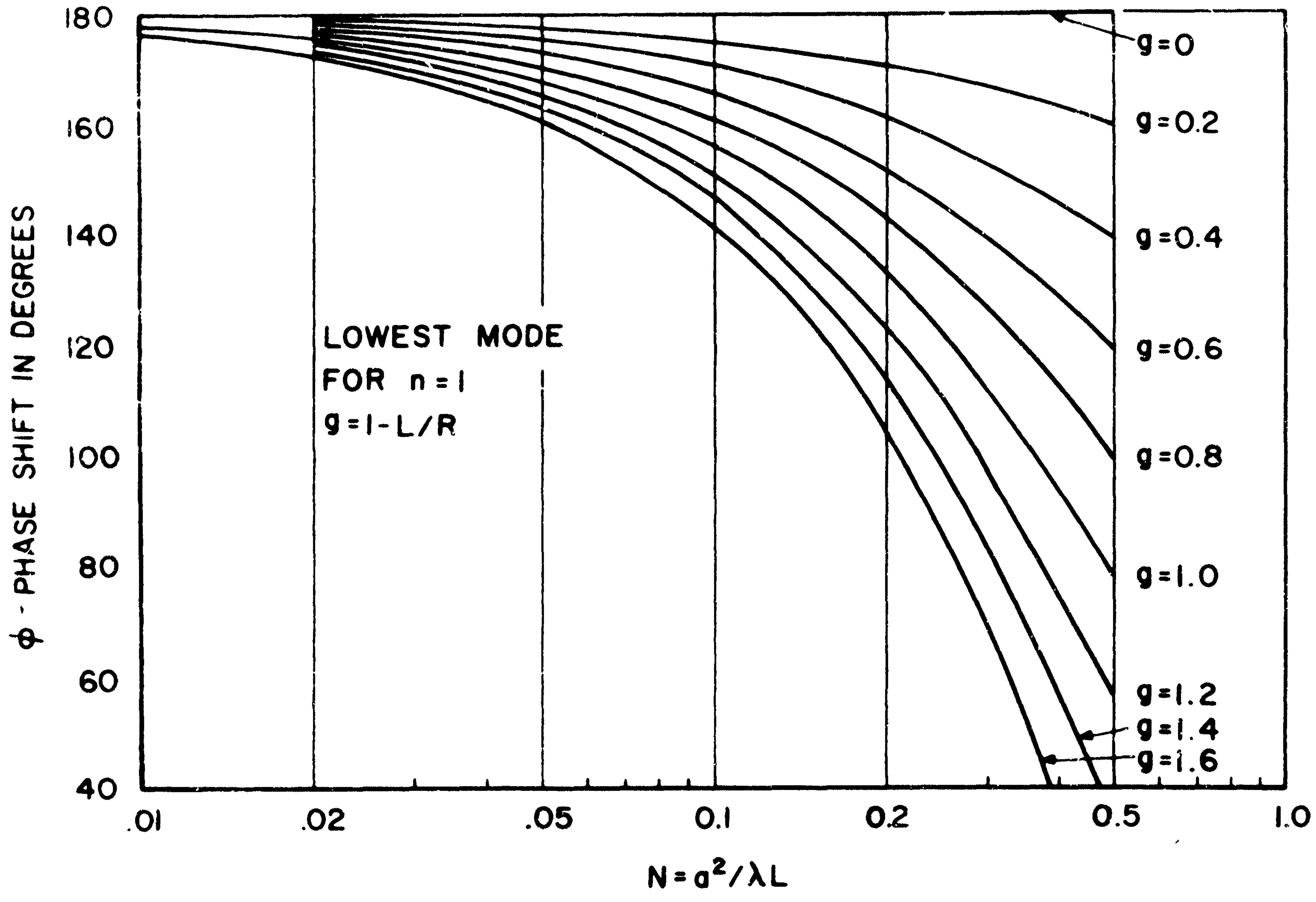

FIGURE 5

PHASE SHIFT PER TRANSIT (RELATIVE TO GEOMETRICAL PHASE SHIFT) vs. FRESNEL NUMBER (LOWEST MODE FOR n=1)

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13. ABSTRACT

A theory of Fabry-Perot resonances designed to be useful at small Fresnel numbers is given. This theory is applied to the case where the interferometer mirrors are of arbitrary curvature and have circular apertures. Asymptotic formulas are derived for the diffraction output and phase shifts in the limit of small Fresnel numbers. These formulas demonstrate the mode discrimination properties of the interferometers when operated in this regime. Numerical results are presented covering a wider range of Fresnel numbers.

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	ROLE	WT	ROLE	WT	ROLE	WT
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